Further space-time correlations of velocity in a turbulent boundary layer

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SUMMARY

This paper describes the results of further experimental investigation of the turbulent boundary layer with zero pressure gradient. Measurements of autocorrelation and of space-time double correlation have been made respectively with single hot-wires and with two hot-wires with the separation vector in any direction. Space-time correlations reach a maximum for some optimum delay. In the case of two points set on a line orthogonal to the plate, the optimum delay T_i is not zero. In the general case it is equal to the corresponding delay T_i, increased by compensating delay for translation with the mean flow. Taylor's hypothesis may be applied to the boundary layer at distances from the wall greater than 3% of the layer thickness. Space-time isocorrelation surfaces obtained with optimum delay have a large aspect ratio in the mean flow direction, even if they are relative to a point close to the wall ($0{\cdot}03\delta$); the correlations along the mean flow then retain high values on account of the large scale of the turbulence.

1. INTRODUCTION

These experiments, following our previous investigations (Favre, Gaviglio & Dumas 1952, 1953 a, b, 1954 c, 1957), deal with longitudinal components of turbulent velocity in the boundary layer on a flat plate, with zero pressure gradient, set in a low turbulence wind tunnel. The plate is made of a glass 210 cm in length, 80 cm in width, with a profiled leading edge and prolonged with an adjustable flap. Transition is obtained by roughness elements beginning at the leading edge.

The velocity outside the boundary layer is 12·20 msec, and the conventional boundary-layer thickness δ is 17·5,33 and 34 mm for distances from the leading edge of 79, 185 and 194 cm (Reynolds numbers R_δ = 14 500, 27 900 and 28 000).

Measurements have been made with hot-wire anemometers, and the apparatus needed for time correlations, autocorrelations, and space-time double correlation coefficients (see Favre 1946, 1948; Favre, Gaviglio & Dumas 1953 a). The space-time correlation coefficient $R_{11}(T, X_1, X_2, X_3)$ for a point A is measured by setting one hot-wire at fixed position A and another at any other position B; the two signals are recorded simultaneously

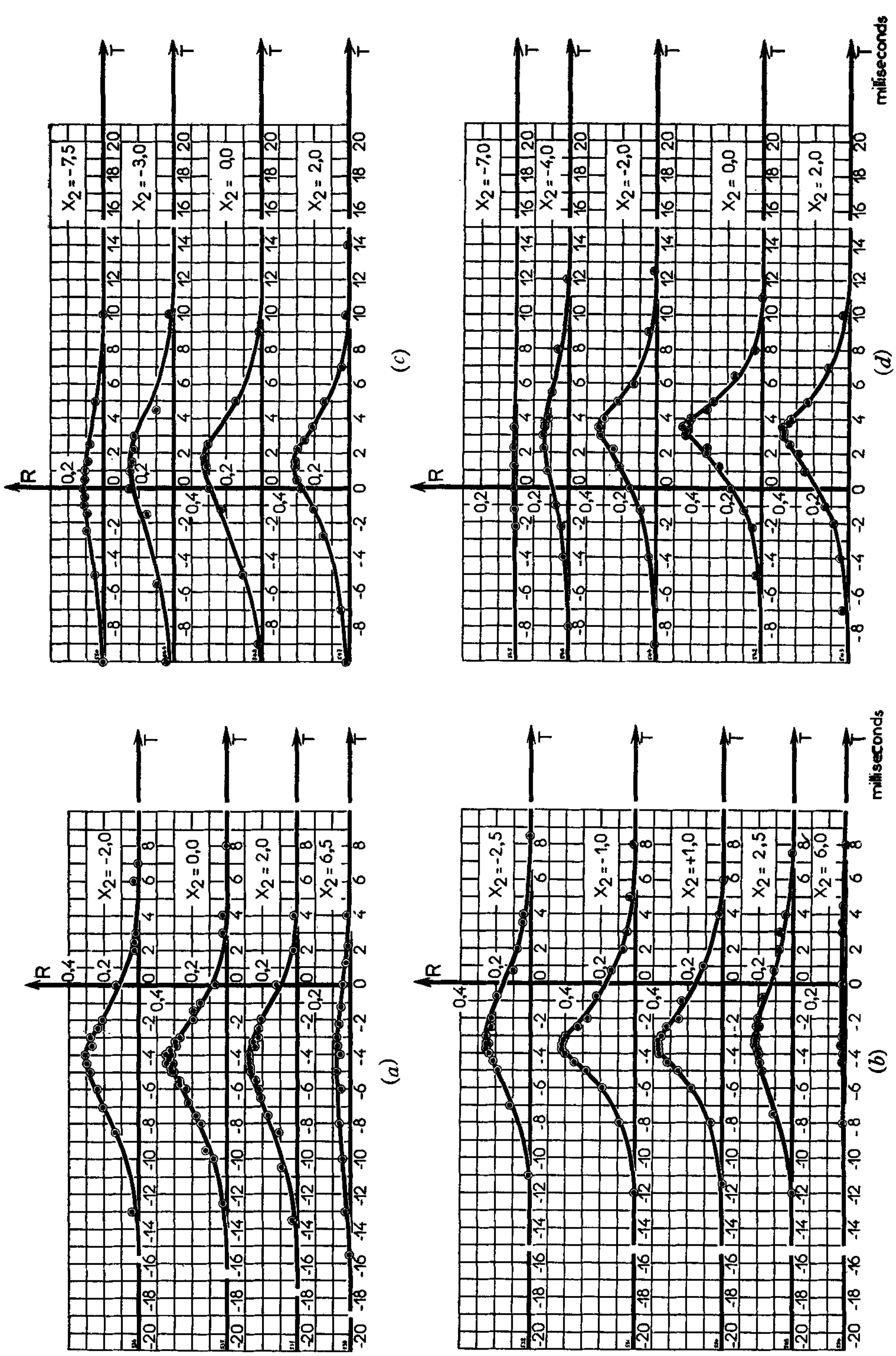

Figure 1. Longitudinal, transverse and lateral space-time correlations in the boundary layer. $\delta = 33$ mm, $R_\delta = 27\,900$, $y' = 1$ mm, X_2 variable. (*a*) $X_1 = -25{\cdot}4$ mm, $X_3 = y' - y = -4$ mm; (*b*) $X_1 = -25{\cdot}4$ mm, $X_3 = 0{\cdot}1$ mm; (*c*) $X_1 = 25{\cdot}4$ mm, $X_3 = -4$ mm; (*d*) $X_1 = 25{\cdot}4$ mm, $X_3 = 0$.

and played back with a relative delay T_0. The autocorrelation coefficient $R(T, 0, 0, 0)$ is obtained when the points A and B are the same (a single hot-wire). y' and y will be used to denote the distances from the wall of wires A and B respectively. X_1 is the component of their separation parallel to the plate (positive in the direction of the mean velocity); $X_3 = y' - y$; X_2 is the separation orthogonal to X_1 and X_3.

The influence on the measurements of the disturbances produced by the upstream probe was checked, and lessened by improvement of probes. With regard to space-time correlations the disturbance reached 13% when the two hot-wires were set on the same mean streamline and close by one another ($X_1/\delta = 0{\cdot}25$ for $y'/\delta = 0{\cdot}77$). No disturbance appeared if the distance X_1/δ was greater than 0·4 for $y'/\delta = 0{\cdot}03$, and 1·5 for $y'/\delta = 0{\cdot}77$; this is not a certain criterion for disturbances to be negligible, but it may be accepted as such, at least if the upstream wire is a little more distant from the wall than the downstream wire.

The mean velocity profiles are in agreement with the accepted shapes, and the universal profile proposed by Clauser (1954).

2. Space-time correlations

The space-time correlation coefficient $R_{11}(T, X_1, X_2, X_3)$ with respect to a point A fixed by its distance y' to the wall and its distance z' to the leading edge, is said to be measured:

- 'longitudinally' when $X_2 = X_3 = 0$ (X_1 and T variable);
- 'transversely' when $X_1 = X_2 = 0$ (X_3 and T variable);
- 'longitudinally and transversely' when $X_2 = 0$ (X_1, X_3 and T variable);
- 'longitudinally, transversely and laterally' when X_1, X_2, X_3 and T are variable.

Numerous measurements were made; figure 1 represents as an example some values of space-time correlations measured longitudinally, transversely and laterally with regard to a fixed point A ($y' = 1$ mm; $z = 185$ cm). It is found that the correlation R_{11} changes with delay T and reaches a maximum for an optimum value of delay to be denoted by T_m in general and by T_i in the particular case of transverse correlations.

Optimum delay T_i for transverse space-time correlation $R_{11}(T, 0, 0, X_3)$

Figures 2 and 3 represent $(V_{y'} - V_y)T_i/\delta$ ($V_{y'}$ and V_y are mean velocities at the positions A and B) measured when the transition is obtained by preturbulence (Favre, Gaviglio & Dumas 1954a) and by roughness respectively; the differences are small. Comparison at distances $z = 79$ cm and $z = 194$ cm (the difference being about 45 times the mean thickness of boundary layer) shows that for the same values of y'/δ and y/δ the delay T_i is proportional to δ. It will be noticed that the variation of T_i as a function of z is very small by comparison with the time required for movement with the mean flow over the same range of z. Figure 4 shows values of T_i for other distances y'/δ of fixed wire from the wall.

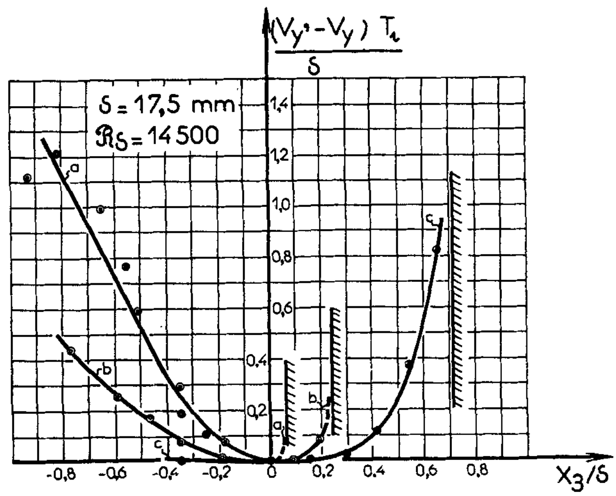

Figure 2. Optimum delay T_i for transverse space-time correlations in the boundary layer on a flat plate (transition by preturbulence).

	y'/δ	Z	curve
⊡	0·06	74 cm	*a*
⊙	0·06	79 cm	*a*
◎	0·24	79 cm	*b*
⊛	0·71	79 cm	*c*

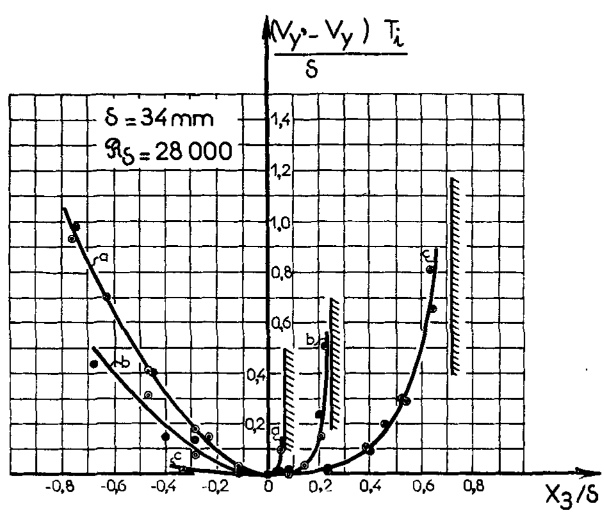

Figure 3. Optimum delay T_i for transverse space-time correlations in the boundary layer on a flat plate (transition by roughness).

	y'/δ	Z	curve		y'/δ	Z	curve
⊙	0·06	194 cm	*a*	⊡	0·06	79 cm	*a*
◎	0·24	194 cm	*b*	▣	0·24	79 cm	*b*
⊛	0·71	194 cm	*c*	⊽	0·71	79 cm	*c*

Optimum delay T_m for transverse and longitudinal space-time correlations $R_{11}(T, X_1, 0, X_3)$

As previously (Favre, Gaviglio & Dumas 1954b, 1957), we verified in these investigations (especially at distances $X/\delta = 4$) that, as a first approximation, the space-time correlation of velocity fluctuations at the fixed point A and at another point B reaches a maximum for a delay T_m equal to the 'initial' optimum delay T_i (which gives maximum transverse space-time correlation for a point C set on the mean streamline passing

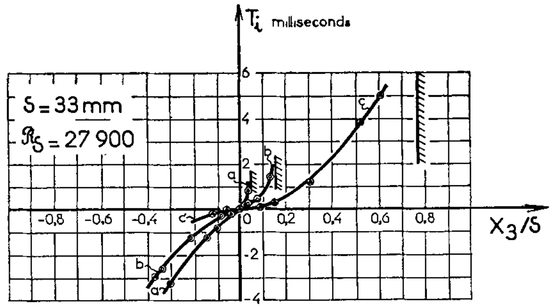

Figure 4. Optimum delay T_i for transverse space-time correlation in the boundary layer on a flat plate (transition by roughness). $Z = 185$ cm.

curve :	*a*	*b*	*c*
$y'/\delta =$	0·03	0·15	0·77

through the point B, with $X_1 = 0$) increased by a delay T_c to compensate for the translation due to the mean flow; i.e.

$$T_m \doteqdot T_i + T_c, \qquad \text{where } T_c \doteqdot \frac{2X_1}{V_A + V_B}.$$

Then, when we pick out several positions B', B'' for point B on the same mean streamline, values of T_m differ from each other by an amount

$$2\left(\frac{CB'}{V_A + V_{B'}} - \frac{CB''}{V_A + V_{B''}}\right) \doteqdot 2\frac{B''B'}{V_A + V_B},$$

since streamlines are close to lines of equal velocity.

We have published (Favre, Gaviglio & Dumas 1957) values of T_m measured and computed from the preceding relation for $y'/\delta = 0{\cdot}06$ in the case of transition obtained by preturbulence, and for $y'/\delta = 0{\cdot}24$ in the case of transition obtained by roughness elements. In figure 5 are given values of T_m corresponding to y'/δ equal to 0·03, 0·15 and 0·77. The above relation is found to be satisfactory to a first approximation when the upstream wire is a little more distant from the wall than the downstream wire* ($X_1X_3 > 0$). Yet this relation is lacking in accuracy when the upstream wire is closer to the wall than the downstream wire ($X_1X_3 < 0$). This may be due to the disturbances produced by the upstream probe.

* Except in the case corresponding to $y'/\delta = 0{\cdot}03$, $X_1 = 76{\cdot}2$ mm, for which the accuracy of measurement of T_m is poor.

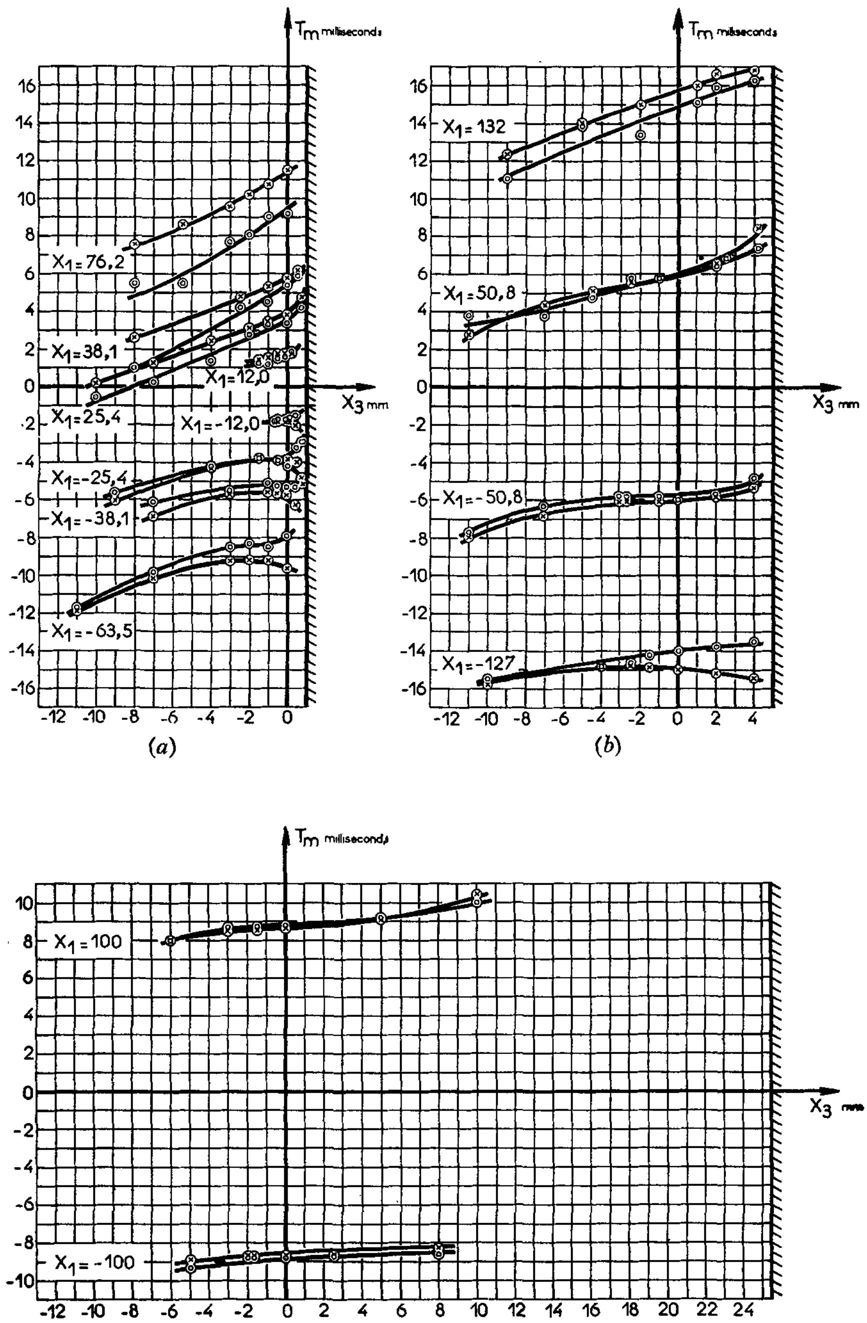

Figure 5. Optimum delays T_m for longitudinal and transverse space-time correlations. $X_2 = 0$, X_1 variable. (*a*) $y'/\delta = 0{\cdot}03$, (*b*) $y'/\delta = 0{\cdot}15$, (*c*) $y'/\delta = 0{\cdot}77$.
⊚ T_m measured, ⊗ T_m computed.

Autocorrelation $R_{11}(T, 0, 0, 0)$ *and longitudinal space correlations* $R_{11}(0, X_1, 0, 0)$; *Taylor's* (1938) *hypothesis*

The previous experiments (Favre, Gaviglio & Dumas 1957) have shown Taylor's hypothesis to be in very satisfactory agreement with autocorrelation curves and longitudinal space correlation curves for the turbulence downstream of a grid of mesh size M when the transformation $T = X_1/V$ is used, at least up to distances beyond which the correlation becomes too small for a conclusive comparison ($X_1/M > 1{\cdot}5$). An analogous comparison has been made for points inside the boundary layer at several distances from the wall, viz. $y'/\delta = 0{\cdot}77$, $0{\cdot}24$*, $0{\cdot}15$ and $0{\cdot}03$ (see figure 6).

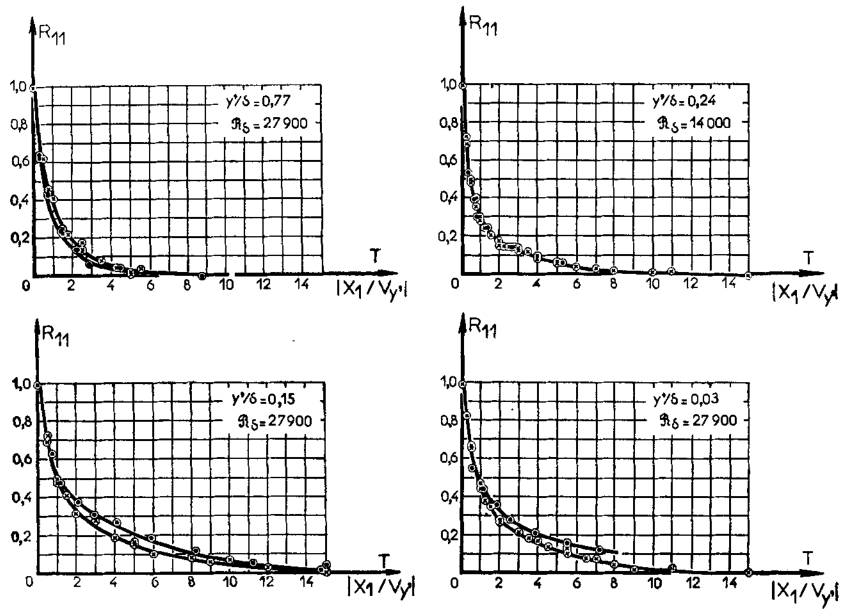

Figure 6. Autocorrelations and longitudinal space correlations; Taylor's hypothesis. ⊗ autocorrelations, ⊙ longitudinal space correlations.

The longitudinal space correlation curves (the streamlines are almost parallel to the plate) are seen to lie close to the autocorrelation curves for $y'/\delta \geqslant 0{\cdot}03$ when the transformation $T = X_1/V_{y'}$ is used. The differences noticed, negligible in the central part of boundary layer ($y'/\delta = 0{\cdot}24$), have opposite signs on the two sides of this zone.

It seems that to a first approximation Taylor's hypothesis may be applied to the turbulent boundary layer, for distances from the wall greater than about 6% of the boundary layer thickness.

Space-time correlations with optimum delay

We have given in a previous paper (Favre, Gaviglio & Dumas 1957) longitudinal and transverse space-time isocorrelation curves $R_{11}(T_m, X_1, 0, X_3)$

* In this case transition is obtained by preturbulence, but the intermittency factor is negligible.

measured with optimum delay T_m at distances y'/δ of the fixed wire A from the wall equal to 0·06, 0·24 and 0·71, and also at $y'/\delta \geqslant 1$ in the case of measurements outside the boundary layer downstream of a grid producing the preturbulence.

When X_1 is given, the space-time correlation $R_{11}(T_m, X_1, 0, X_3)$ with optimum delay T_m reaches a ***maximum maximorum*** for one value of X_3. Corresponding values of X_1 and X_3 determine the 'maximum space-time line with optimum delay' or 'maximum correlation line' that goes through the fixed point A. The present experiments were made in the case of transition obtained by roughness and extended to three space dimensions, longitudinally, transversely, and laterally.

In figures 7, 8, 9 we show space-time isocorrelation surfaces with optimum delay for positions of the fixed point A defined by $y'/\delta = 0{\cdot}03$, 0·15 and 0·77 respectively ($z = 185$ cm). The symmetry with respect to X_2 having been checked, we may define as above the line of maximum correlation corresponding to each position of the fixed point A. Figure 10 gives the space-time correlation coefficients with optimum delay T_m, along the lines of maximum correlations, for positions of the fixed point A defined by $y'/\delta = 0{\cdot}03$, 0·15 and 0·77. These measurements were made with band-passes extending from 1 to 2500 Hz, and from 1 to 275 Hz by cutting off high frequencies.

Space-time isocorrelation surfaces with optimum delay relating to a fixed point A have a great aspect ratio along the mean flow upstream and downstream, even at a small distance of the fixed point A from the wall ($y'/\delta = 0{\cdot}03$). They are symmetrical with respect to X_2, and their dimensions are of the same order transversely as laterally. The lines of maximum correlation outside the boundary layer are the same as the mean streamlines, but they differ markedly inside the boundary layer.

y'/δ	1	0·77	0·71	0·24	0·15	0·06	0·03
Transition by preturbulence $R_\delta = 14\,000$ (band-pass 1–2500 Hz)	7·5 to 12		$\doteqdot$8·5	2·4		1·2	
Transition by roughness $R_\delta = 27\,900$ (band-pass, 1–2500 Hz)		>7			2·1		0·65
Transition by roughness $R_\delta = 27\,900$ (band-pass, 1–275 Hz)		>10			3·6		1·6

Table 1. Values of X_1/δ at which $R_{11} = 0{\cdot}5$.

The correlation coefficient with optimum delay retains high values for distances X_1/δ along the mean flow upstream and downstream which are appreciable at $y'/\delta \geqslant 0{\cdot}03$ and increase with y'/δ. As an example, the value $R_{11} = 0{\cdot}5$ is preserved as far as the distances indicated in table 1.

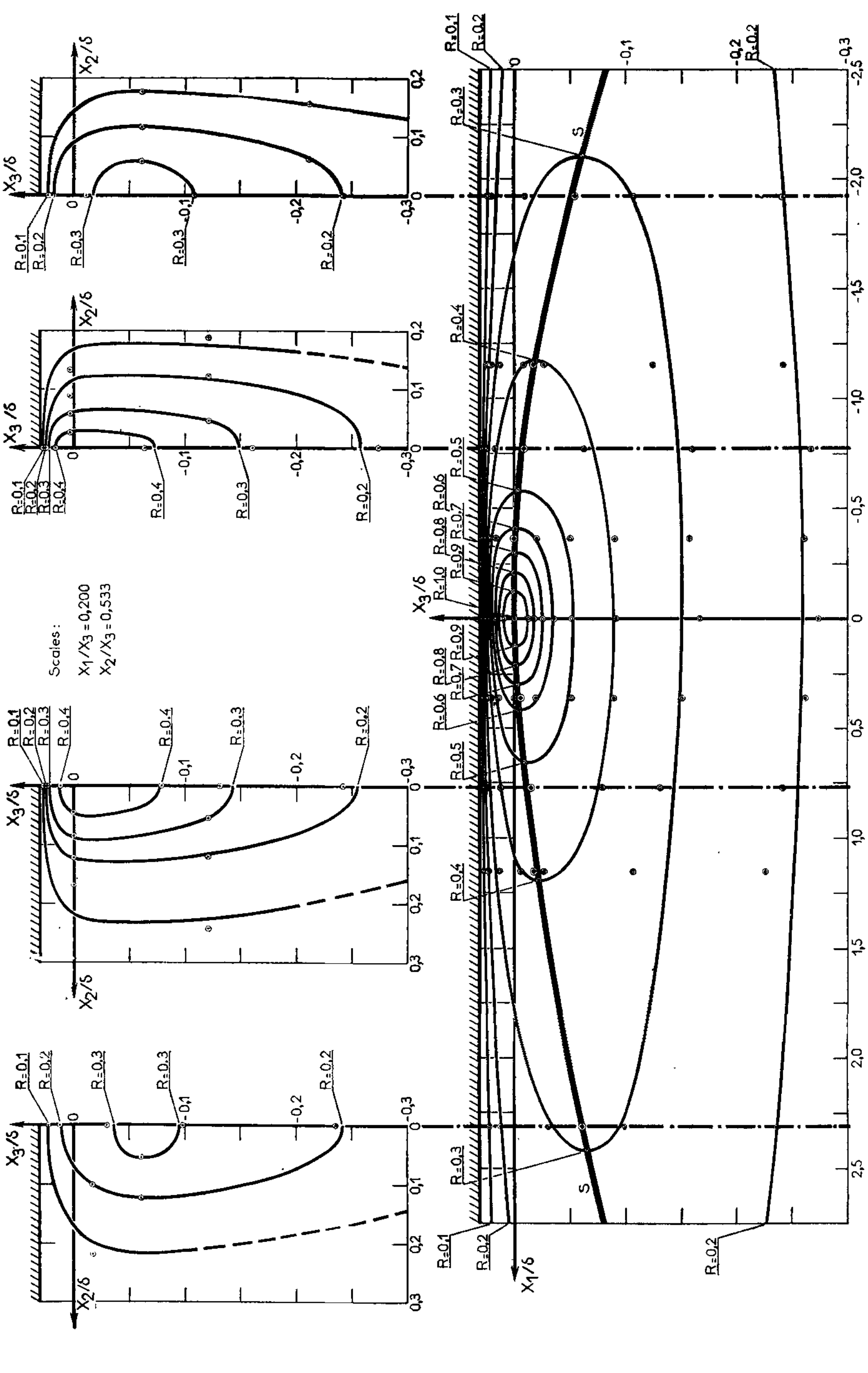

Figure 7. Space-time isocorrelation surfaces with optimum delay in the boundary layer on a flat plate; $\delta = 33$ mm, $R_\delta = 27900$, $y'/\delta = 0{\cdot}03$.

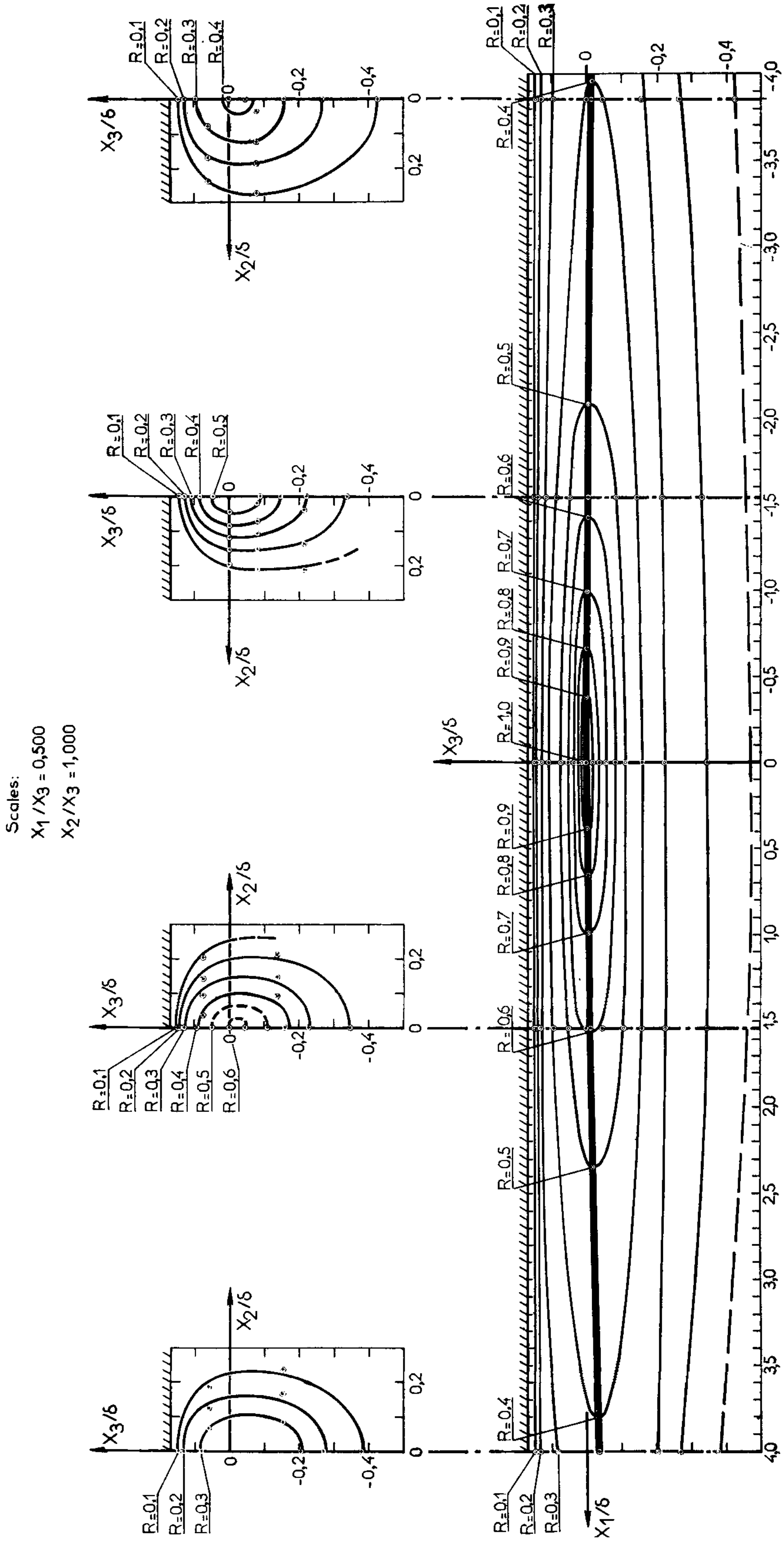

Figure 8. Space-time isocorrelation surfaces with optimum delay in the boundary layer on a flat plate; $\delta = 33$ mm, $R_\delta = 27\,900$, $y'/\delta = 0{\cdot}15$.

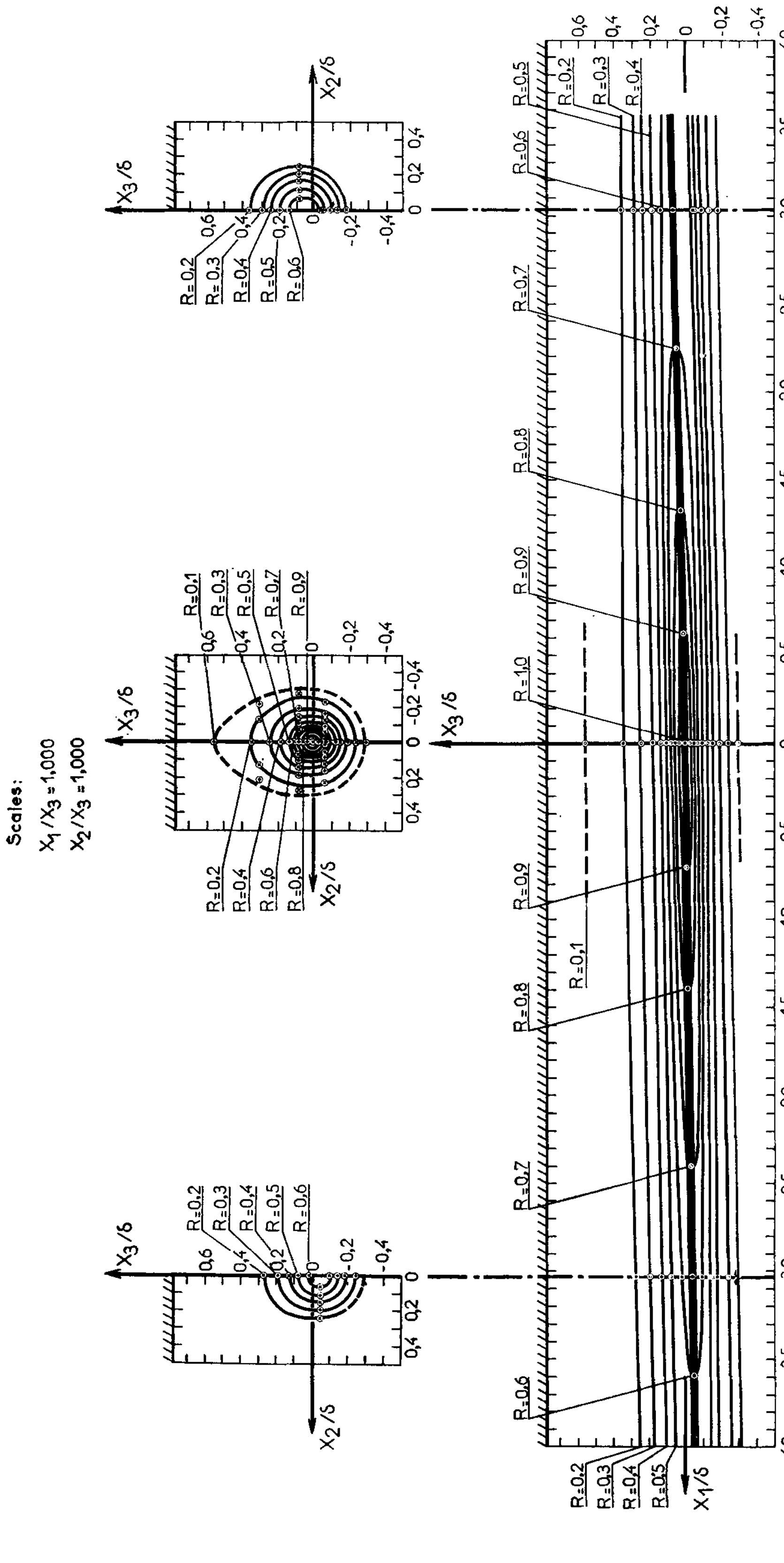

Figure 9. Space-time isocorrelation surfaces with optimum delay in the boundary layer on a flat plate; $\delta = 33$ mm, $R_\delta = 27\,900$, $y'/\delta = 0.77$.

The distances are increased markedly by cutting off frequencies higher than 275 Hz, a result similar to that concerning the turbulence behind a grid (Favre, Gaviglio & Dumas 1954c).

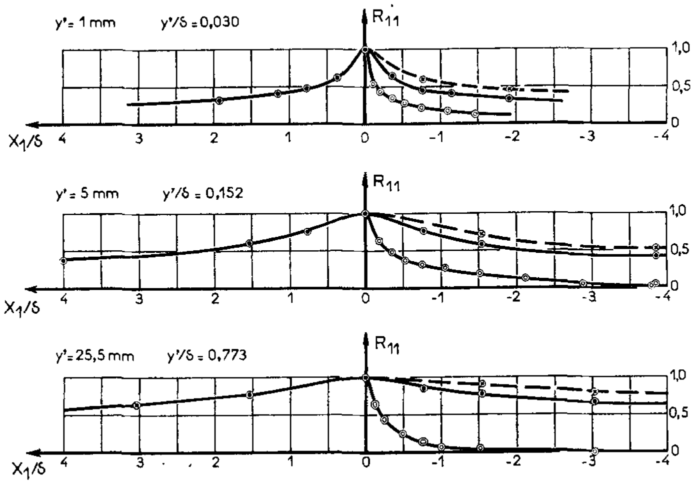

Figure 10. Space-time correlations in the boundary layer on a flat plate.

◉ Along the line of maximum correlation, with optimum delay T_m, band-pass 1–2500 Hz.

⊗ Along the line of maximum correlation, with optimum delay T_m, band-pass 1–275 Hz.

◎ Along a mean streamline, with zero delay.

3. Conclusions

The space-time correlation reaches a maximum for an optimum delay T_m. When several alternative positions B', B'' on a mean streamline are chosen for one of the points (B), the values of T_m differ from each other by a compensating delay due to the motion from B' to B'' with the mean flow, the effective velocity of translation being taken as the average of the mean flow velocity at the point B and at the other point A.

The values of the 'initial' optimum delay (corresponding to the case of the two points being set orthogonally to the plate) noted at two stations for which the distances from the leading edge are very different (the distances of the points from the wall being the same fraction of δ) are proportional to the boundary layer thickness δ. Variation of T_i with x is very small compared with the times corresponding to translation with the mean flow over the same range of x. This seems to be due to the renewal of turbulence between the two distant positions.

Taylor's hypothesis may be applied, as a first approximation, to the boundary layer, for distances from the wall greater than about 6% of the thickness δ.

Space-time isocorrelation surfaces, with optimum delay relative to a fixed point A, have a great aspect ratio along the mean flow, upstream and downstream, even for small distances of the fixed point from the wall

(3% of δ). Their dimensions are of the same order transversely and laterally. The maximum correlation lines and mean streamlines are different inside the boundary layer, but are coincident outside.

The correlation coefficient with optimum delay retains high values for distances upstream and downstream which are significant for $y'/\delta = 3\%$ and increase in proportion to the distance of this fixed point from the wall. These distances are increased by cutting off the high frequency components.

Along the mean streamlines the intensity of turbulence is practically constant and the correlation with optimum delay shows, on the one hand, inasmuch as it retains high values, the long persistence of large scale turbulence translated by the mean motion, and, on the other hand, in the light of its diminution, the evolution of the turbulence and its renewal inside the boundary layer.

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